Nuclear equation of state from ground and excited state properties of nuclei

3.- Current status on the determination of the nuclear Equation of State

Xavier Roca-Maza
Università degli Studi di Milano
INFN sezione di Milano

École Joliot-Curie
“Nuclear Matter under Pressure”
Saint Pierre d’Oleron, France
September 4th-9th

What can we learn from the Earth and the Heavens about the Nuclear Equation of State?

(some examples)

From Heaven: Neutron Star Mass

Nuclear models that account for different nuclear properties on **Earth** predict a large **variety** of **Neutron Star Mass-Radius** relations → **<u>Observation of a $2M_{sun}$ has constrained nuclear models.</u>**

Tolman–Oppenheimer–Volkoff equation (sph. sym.):

$$\frac{dM(r)}{dr} = 4\pi r^2 \mathcal{E}(r);$$

$$\frac{dP}{dr} = -G\frac{\mathcal{E}(r)M(r)}{r^2}\left[1+\frac{P(r)}{\mathcal{E}(r)}\right]\left[1+\frac{4\pi r^3 P(r)}{M(r)}\right]\left[1-\frac{2GM(r)}{r}\right]^{-1}$$

$\mathcal{E}(r) \rightarrow$ degeneracy pressure from neutrons $\rightarrow M_{\max} = 0.7 M_{\rm sun}$

Nuclear Physics input is fundamental

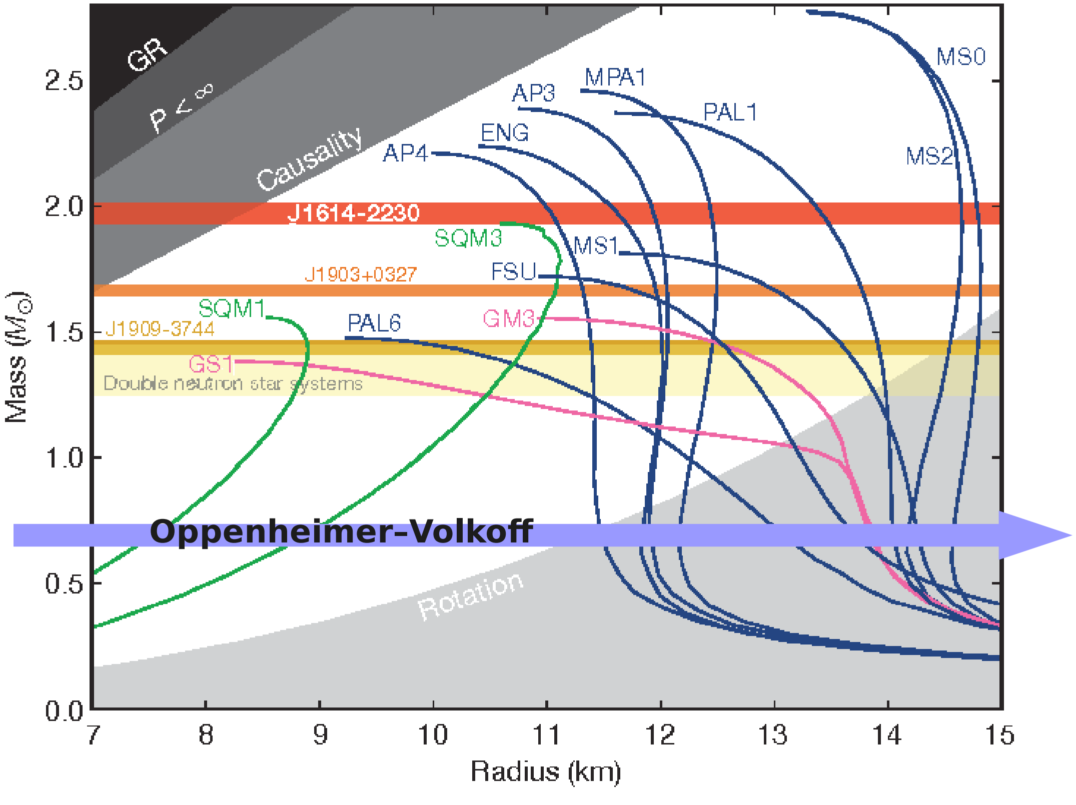


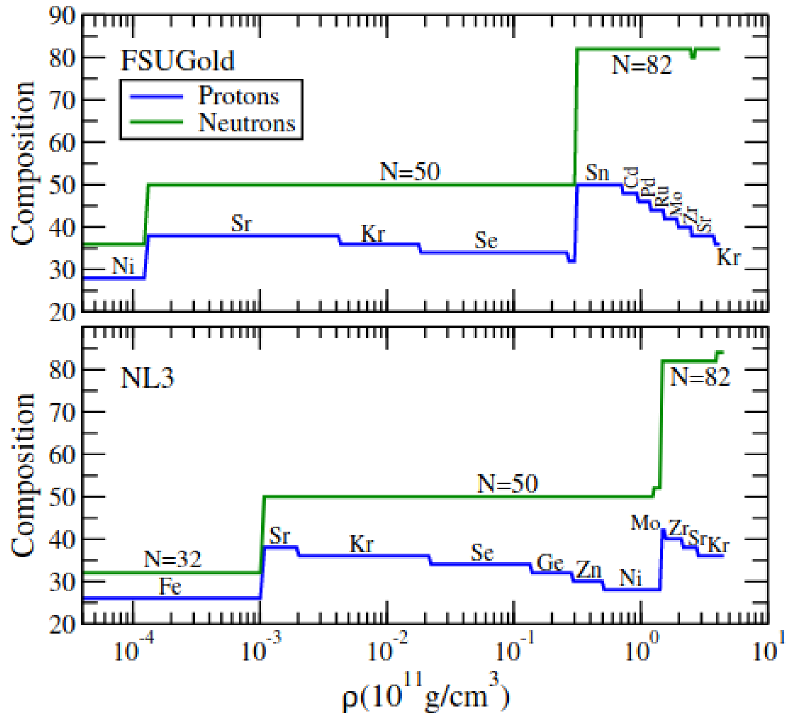
Figure 3 | Neutron star mass–radius diagram. The plot shows non-rotating

A two-solar-mass neutron star measured using Shapiro delay - P. B. Demorest, T. Pennucci, S. M. Ransom, M. S. E. Roberts & J. W. T. Hessels - Nature volume 467, 1081–1083(2010)

From Heaven: outer crust composition

- → span 7 orders of magnitude in **denisty** (from **ionization** $\sim 10^4$ g cm to the **neutron drip** $\sim 10^{11}$ g cm)
- → it is organized into a **Coulomb lattice** of neutron-rich nuclei (ions) embedded in a relativistic **uniform electron gas**
- → $T \sim 10^6$ K ~ 0.1 keV → one can treat **nuclei and electrons at $T = 0$ K**
- → At the **lowest densities**, the electronic contribution is negligible so the Coulomb lattice is populated by ^{56}Fe nuclei.
- → As the **density increases**, the electronic contribution becomes important, it is energetically advantageous to lower its electron fraction by $e^- + (N, Z) \rightarrow (N+1, Z-1) + \nu_e$ and therefore $Z \downarrow$ with constant (approx) number of N
- → As the **density continues to increase, penalty energy from the symmetry energy** due to the neutron excess changes the composition to a dif ferent $N-$*plateau*

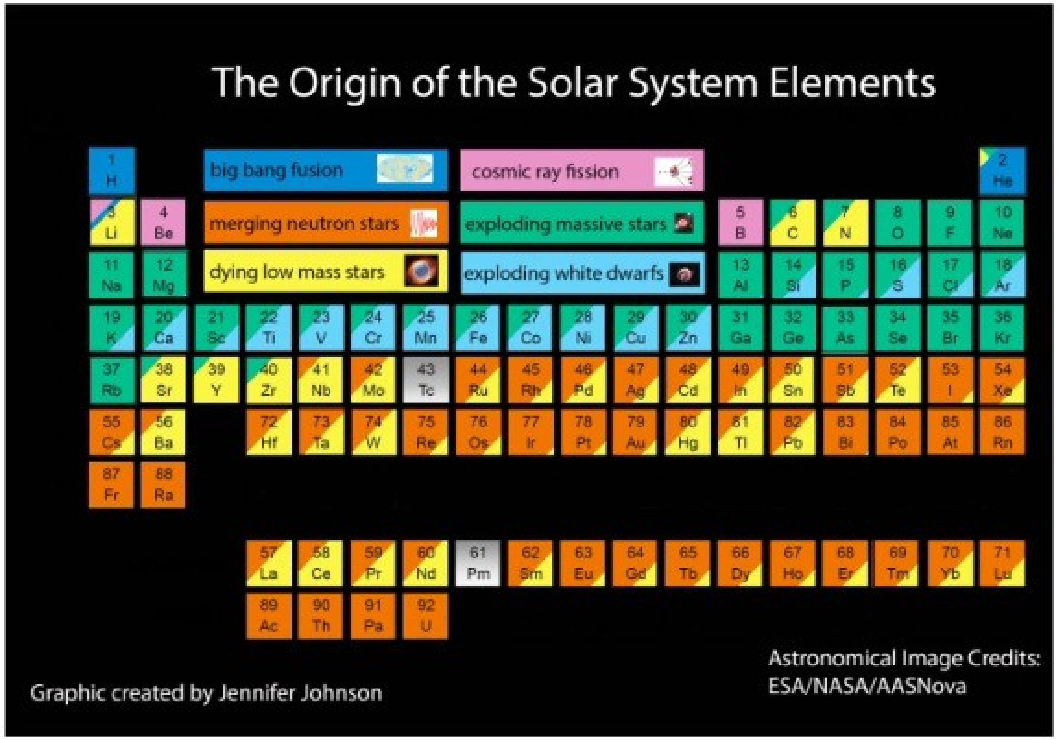
  $$\frac{Z}{A} \approx \frac{Z_0}{A_0} - \frac{p_{Fe}}{8a_{sym}} \quad \text{where } (A_0, Z_0) = {}^{56}\text{Fe}_{26}$$
- → The Coulomb lattice is made of more and more neutron-rich nuclei until the critical **neutron-drip density is reached ($\mu_{drip} = m_n$)**. $[M(N,Z) + m_n < M(N+1, Z)]$



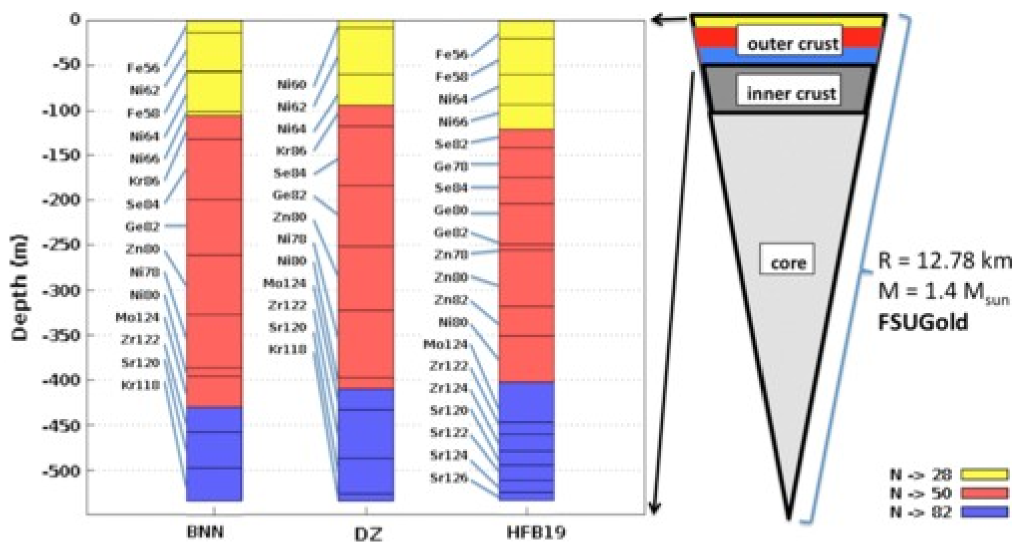
Physical Review C **78**, *025807 (2008)*

The faster the symmetry energy increases with density ($L \uparrow$), the more exotic the composition of the outer crust.

From Heaven: Origin of elements



The **crust** of a **NS** is made of very **exotic** **neutron rich nuclei,** stable only due to the extreme conditions (large densities). **Different nuclear models predict different compositions**

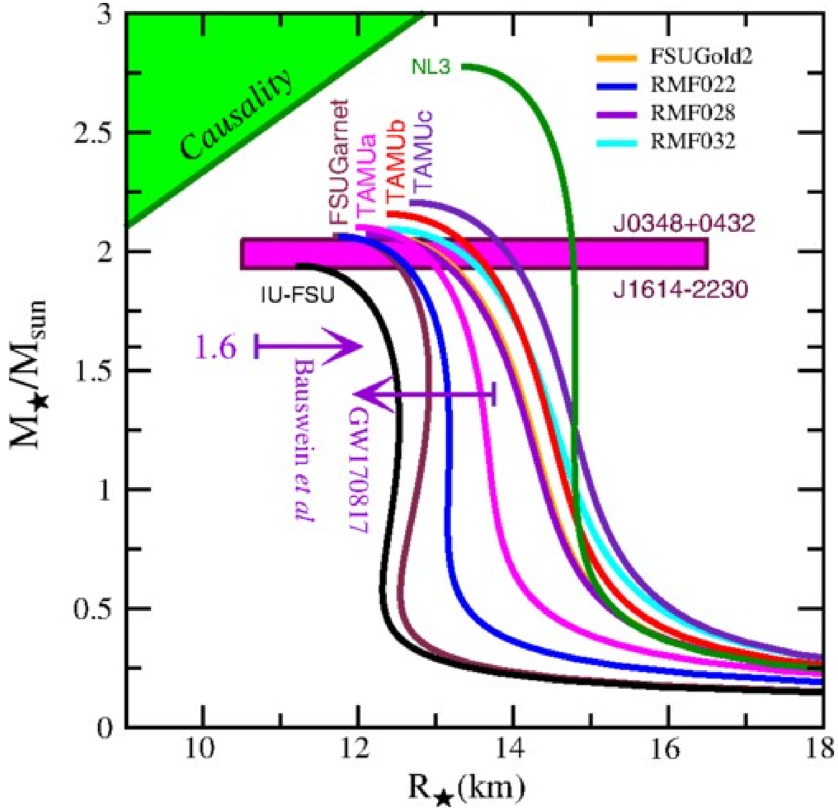


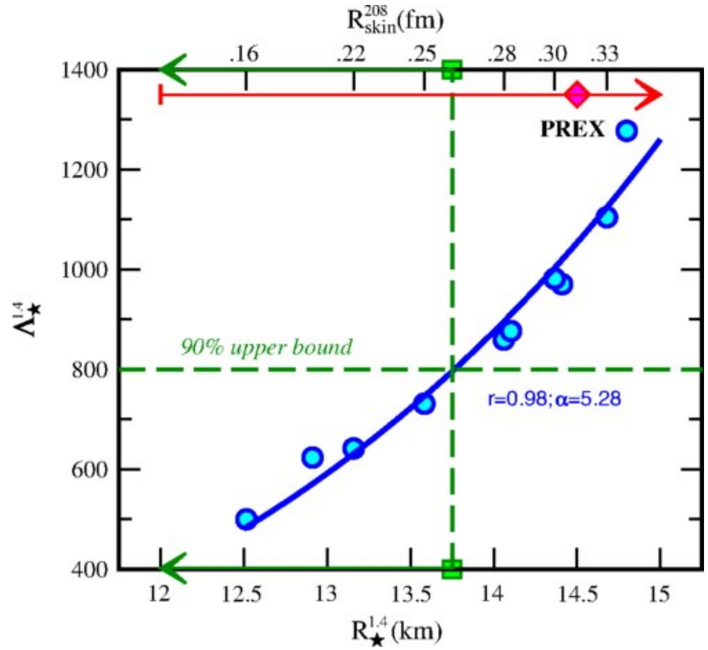
Nuclear mass predictions for the crustal composition of neutron stars: A Bayesian neural network approach R. Utama, J. Piekarewicz, and H. B. Prosper, Phys. Rev. C 93, 014311 (2016)

Binary neutron star merger produced about **10^{29}kg** of **heavy elements**!

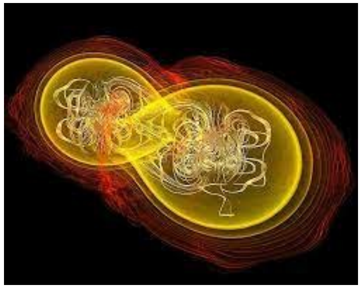
From Heaven: Gravitational wave signal from a binary neutron star merger

GW170817 from the binary neutron star merger → **constraint** neutron star **radius** and, thus, the **nuclear EoS**





Tidal deformability (Λ) is a quadrupole deformation inferred from **GW signal →** proportional to **restoring force.** Hence, sensitive to the **nuclear EoS**



Neutron Skins and Neutron Stars in the Multimessenger Era

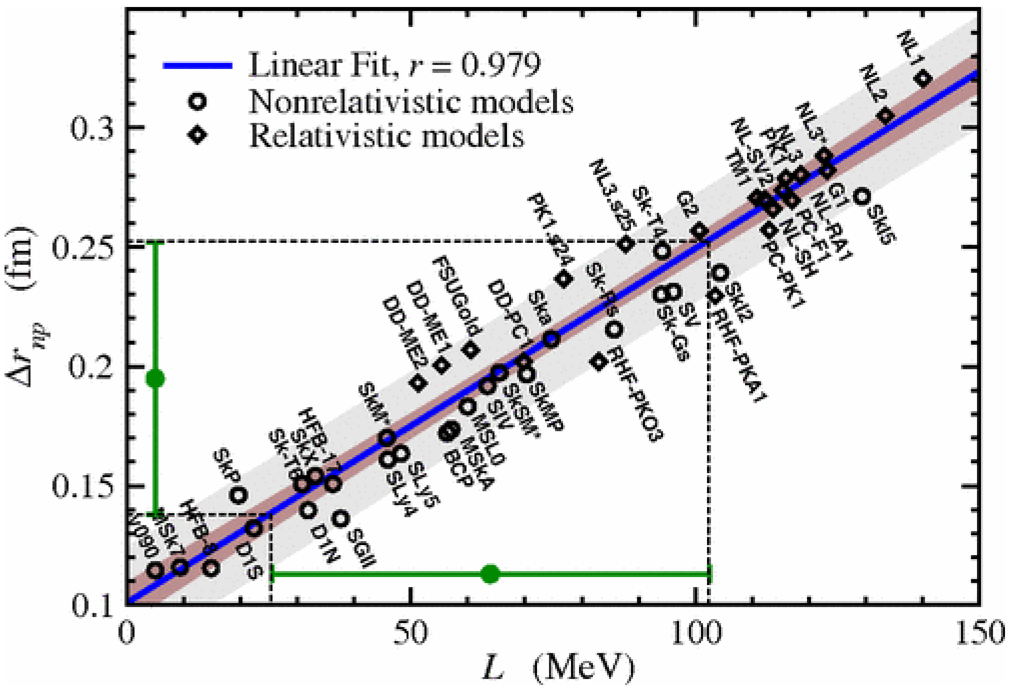
F. J. Fattoyev, J. Piekarewicz, and C. J. Horowitz Phys. Rev. Lett. 120, 172702 (2018)

From Heaven & Earth: neutron skin and the Radius of a Neutron Star

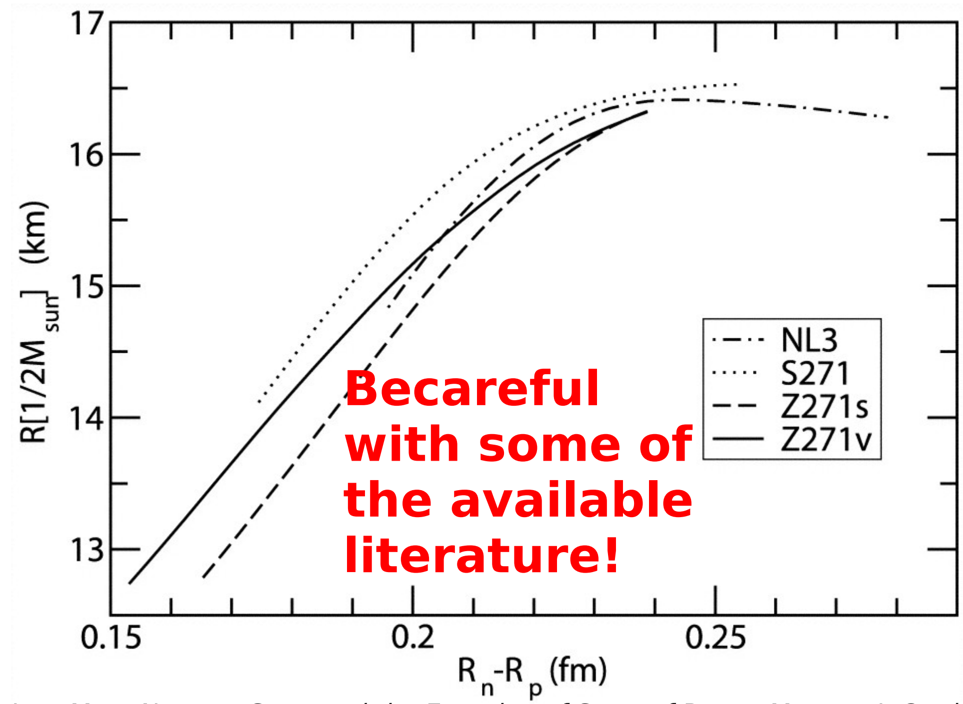
Both, the **neutron skin thickness** ($\Delta r_{np}=r_n-r_p$) in neutron rich nuclei and the **radius** of a **neutron star** are related to the **neutron pressure** in infinite matter. The former around ρ_0 (L) while the latter in a broad range of densities.

$$\Delta r_{np} \approx \frac{1}{12}\frac{N-Z}{A}\frac{R}{J}L$$

→ Only for <u>unrealistically</u> **small neutron stars**, that is, for small central densities ($\rho_c \sim \rho_0$): nuclear models predict a **linear** relation between **R** and **Δr_{np}...**

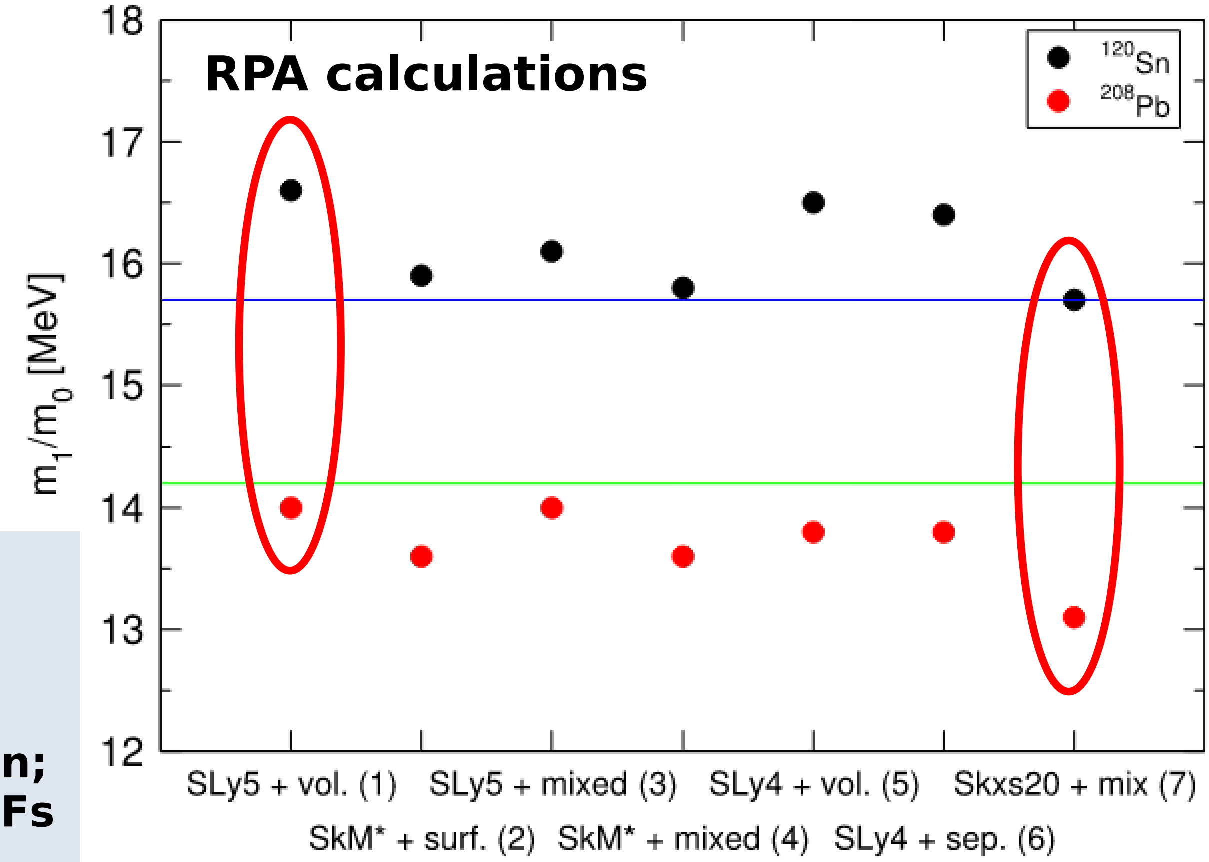


Neutron Skin of 208Pb, Nuclear Symmetry Energy, and the Parity Radius Experiment
X. Roca-Maza, M. Centelles, X. Viñas, and M. Warda Phys. Rev. Lett. 106, 252501 (2011)



Low-Mass Neutron Stars and the Equation of State of Dense Matter - J. Carriere, C. J. Horowitz, and J. Piekarewicz - The Astrophysical Journal, 593 (2003) 463

From Earth: Giant Monopole Resonance
do we understand it?



Exp.

K_0<**240** MeV

(Open shell)

Exp.

K_0=**240±20** MeV

(Closed shell)

Pairing effects studied here

Further studies are needed (uncertainty quantification; Extended EDFs & ab initio)

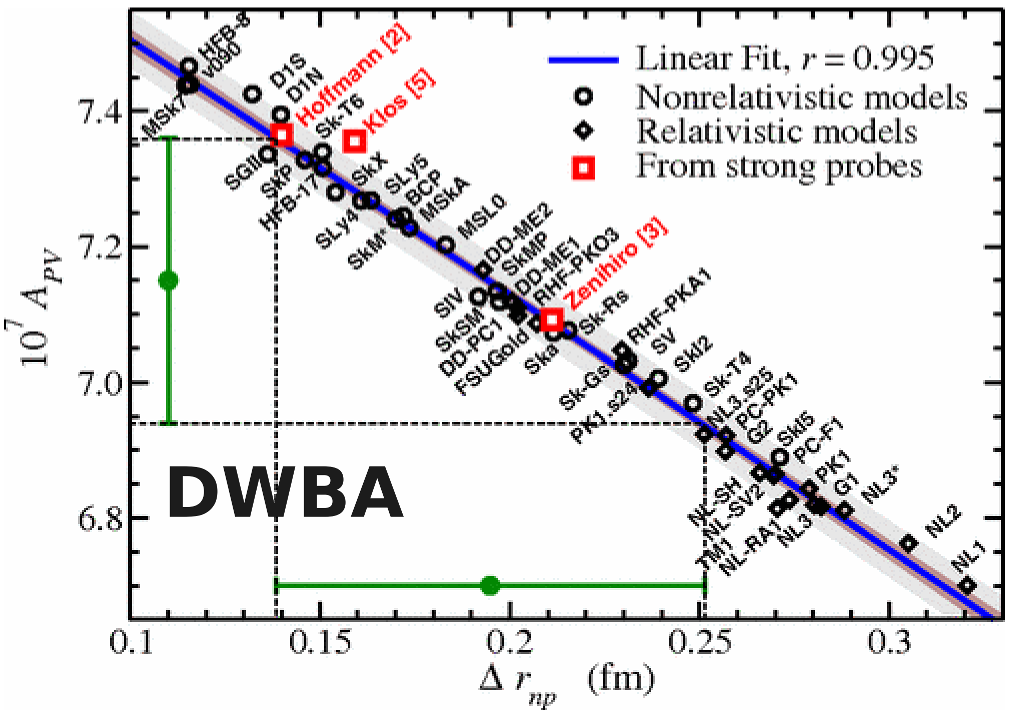
U. Garg, G. Colò / Progress in Particle and Nuclear Physics 101 (2018) 55–95

From Earth: Parity violating electron scattering and the neutron skin

Polarized electron-Nucleus scattering:

→ In good approximation, **the weak interaction** probes the **neutron distribution** in nuclei while **Coulomb interaction** probes the **proton distribution**

→ **Different experimental efforts @ Jlab (USA) & MAMI (Germany)**



Neutron Skin of 208Pb, Nuclear Symmetry Energy, and the Parity Radius Experiment
X. Roca-Maza, M. Centelles, X. Viñas, and M. Warda Phys. Rev. Lett. 106, 252501 (2011)

→ **Electrons** interact by **exchanging** a **γ** (couples to **p**) or a **Z_0** boson (couples to **n**)

→ **Ultra-relativistic electrons**,depending on their helicity (±), will interact with the nucleus seeing a slightly different potential: **Coulomb** ± **Weak**

$$A_{pv} = \frac{d\sigma_+/d\Omega - d\sigma_-/d\Omega}{d\sigma_+/d\Omega + d\sigma_-/d\Omega} \sim \frac{\text{Weak}}{\text{Coulomb}}$$

→ Main **unknown** is **ρ_n**

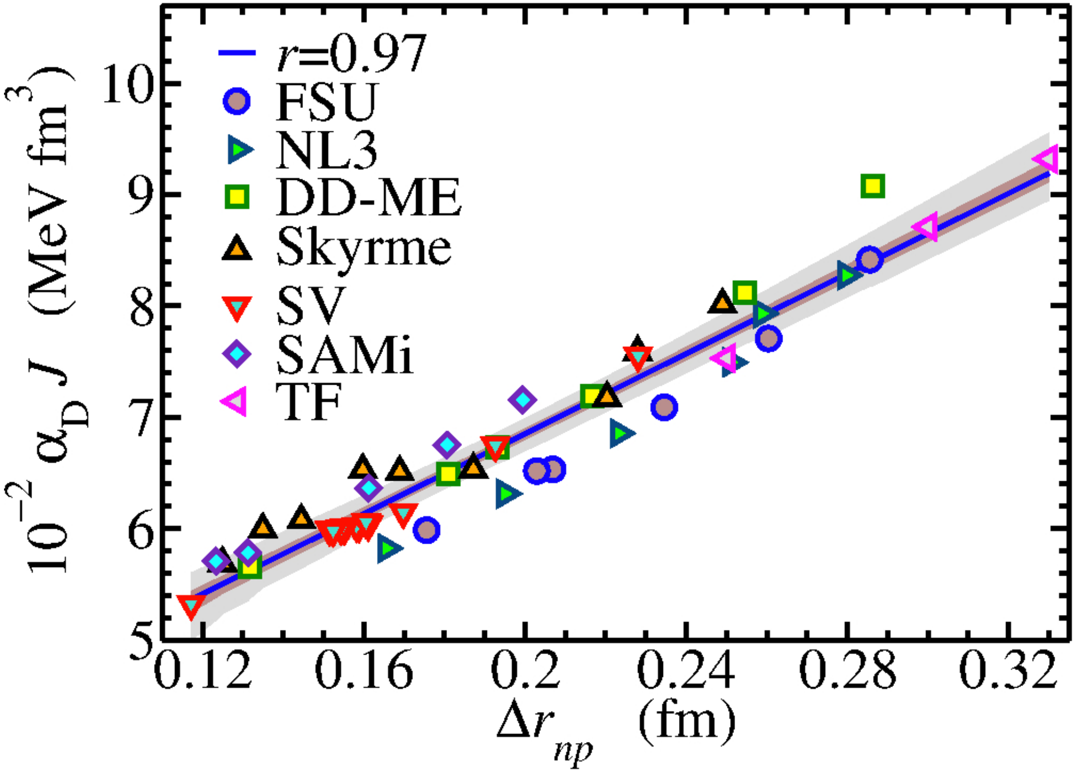
→ In **PWBA** for small momentum transfer **q**:

$$A_{pv} = \frac{G_F q^2}{4\sqrt{2}\pi\alpha}\left(1 - \frac{q^2 r_p^2}{3F_p(q)}\right)\Delta r_{np}$$

From Earth: dipole polarizability and neutron skin

The dipole **polarizability** measures the **tendency** of the nuclear **charge** distribution to be **distorted.**

From a macroscopic point of view **α ~ (electric dipole moment)/(external electric field)**



→ Using the **dielectric theorem**: the polarizability can be computed from the expectation value of the Hamiltonian in the constrained ground state H′=H+λD

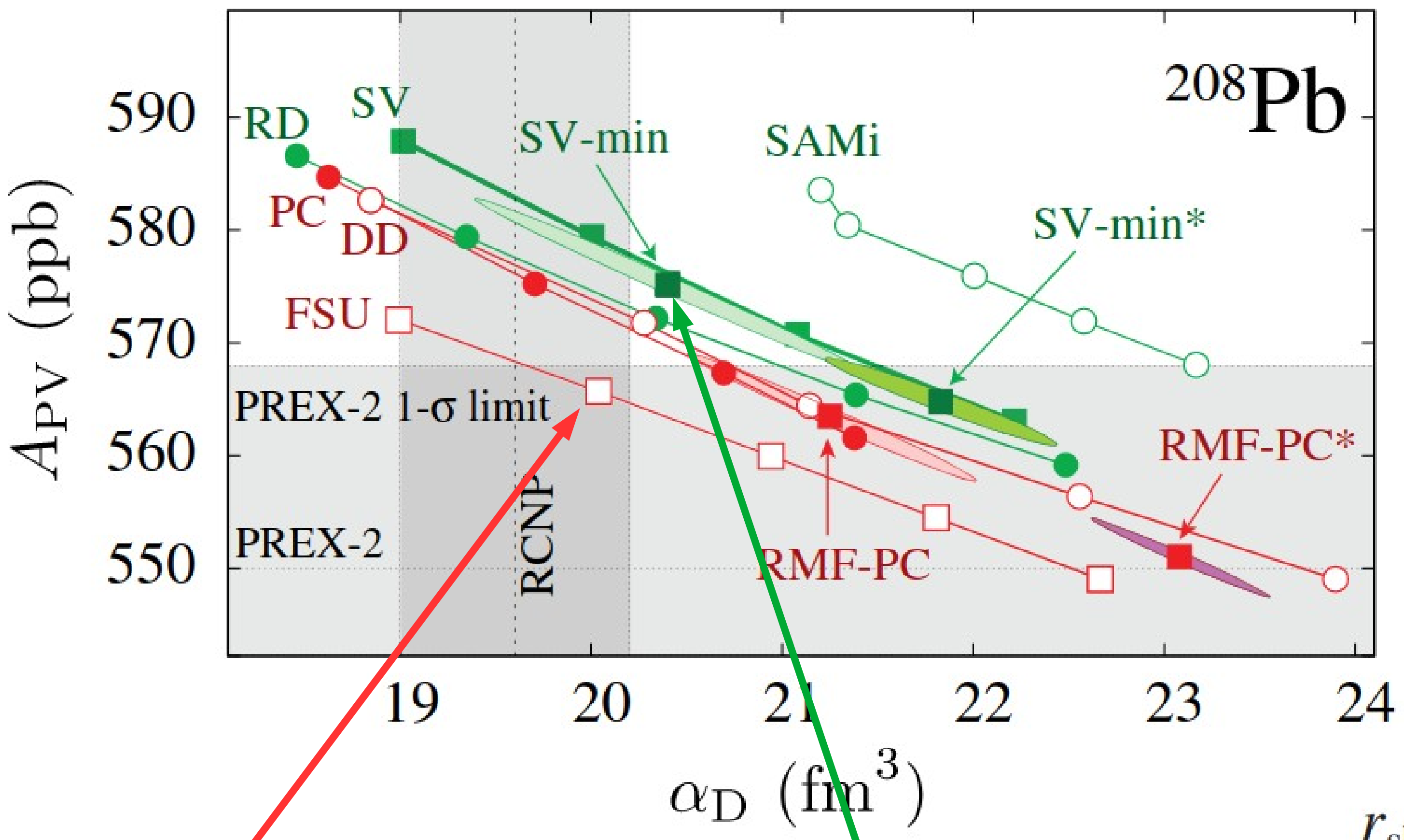
→ For guidance assuming the **Droplet model** for H, one would find:

$$\alpha_D \approx \frac{\pi e^2}{54}\frac{\langle r^2\rangle}{J}A\left(1+\frac{5}{2}\frac{\Delta r_{np}-\Delta r_{np}^{\mathrm{surf}}-\Delta r_{np}^{\mathrm{Coul}}}{\langle r^2\rangle^{1/2}(I-I_{\mathrm{Coul}})}\right)$$

Electric dipole polarizability in 208Pb: Insights from the droplet model - X. Roca-Maza, M. Brenna, G. Colò, M. Centelles, X. Viñas, B. K. Agrawal, N. Paar, D. Vretenar, and J. Piekarewicz

Phys. Rev. C 88, 024316 (2013)

From Earth: A_{PV} (JLab) versus α_D (RCNP)



Paul-Gerhard Reinhard, Xavier Roca-Maza, and Witold Nazarewicz
Phys. Rev. Lett. 127, 232501 (2021)

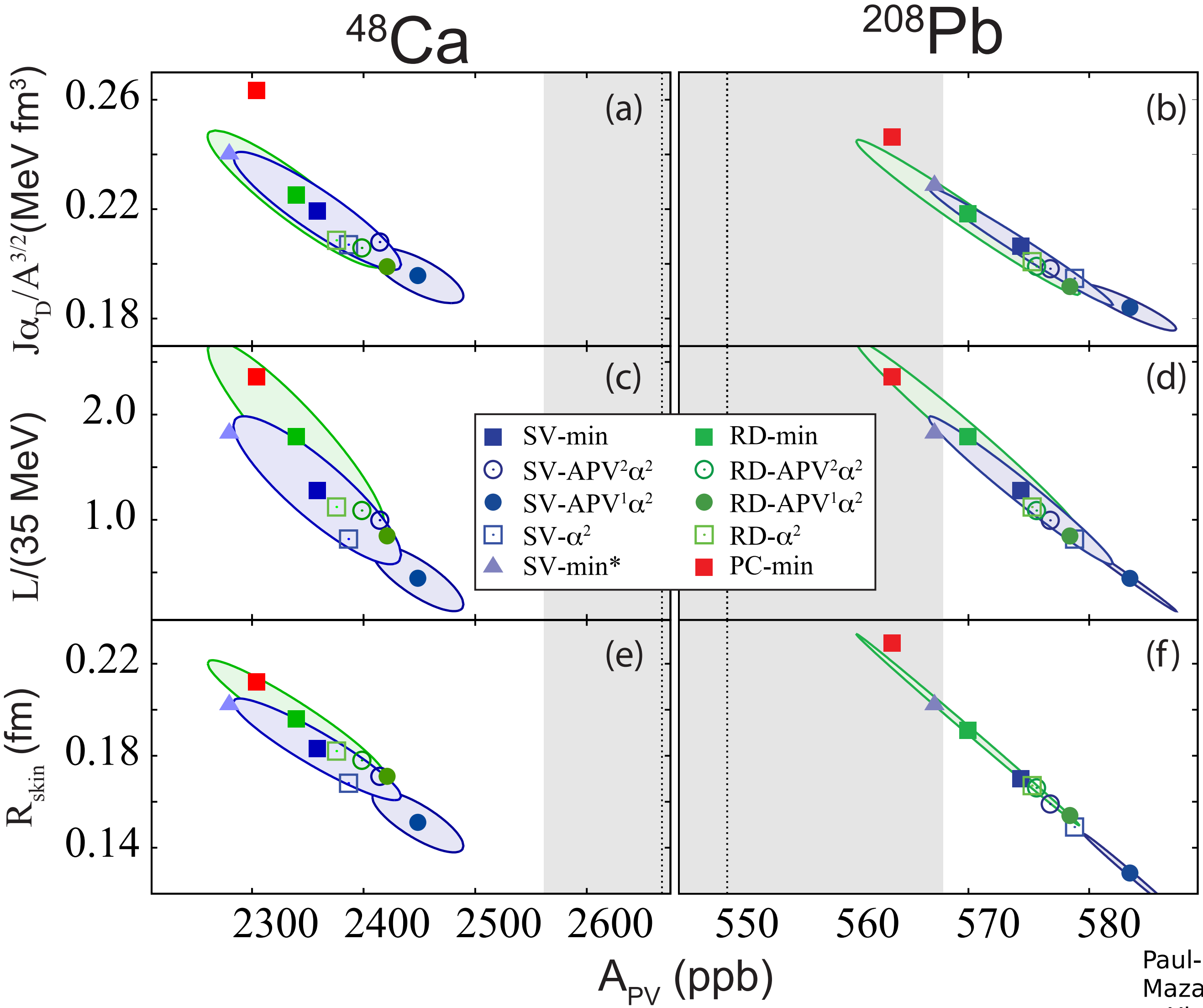
SV-min, suitable model to predict EoS around ρ_0 and Δr_{np}

$$r_{skin} = 0.19 \pm 0.02 \text{ fm}$$
$$L = 54 \pm 8 \text{ MeV}$$

B and **Rch** away from "expected" EDF accuracy for B and Rch

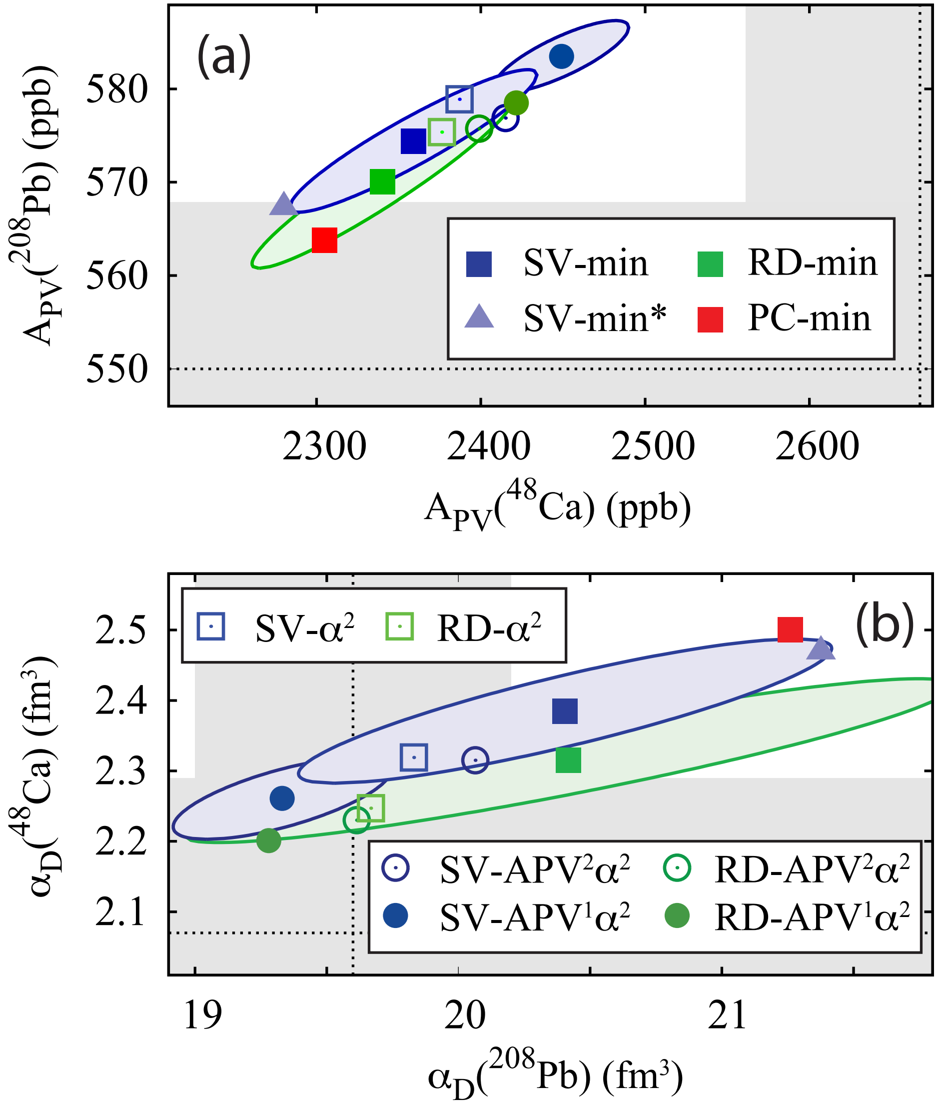
Theoretical and experimental 1σ errors overlap for SV-min
But not overlap both exp.

From Earth: A_{pv} (JLab) versus α_D (RCNP)



Paul-Gerhard Reinhard, Xavier Roca-Maza, and Witold Nazarewicz
arXiv:2206.03134

From Earth: A_{PV} (JLab) versus α_D (RCNP)



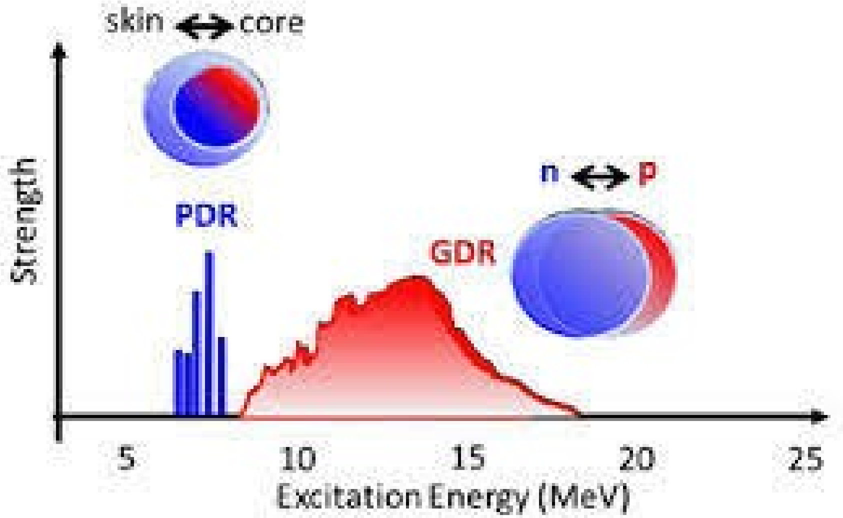
Paul-Gerhard Reinhard, Xavier Roca-Maza, and Witold Nazarewicz
arXiv:2206.03134

From Heaven & Earth: low energy dipole response and nucleosynthesis

The **largest** the **neutron pressure** among neutrons (~L), the more the **excess neutrons** (~skin) are ***"pushed out"*** in the **outermost** part of the **nucleus →** spatial ***decorrelation*** of some of those neutrons with the nucleons in the core produces **larger low lying** responses**.**

GDR=Giant Dipole Resonance

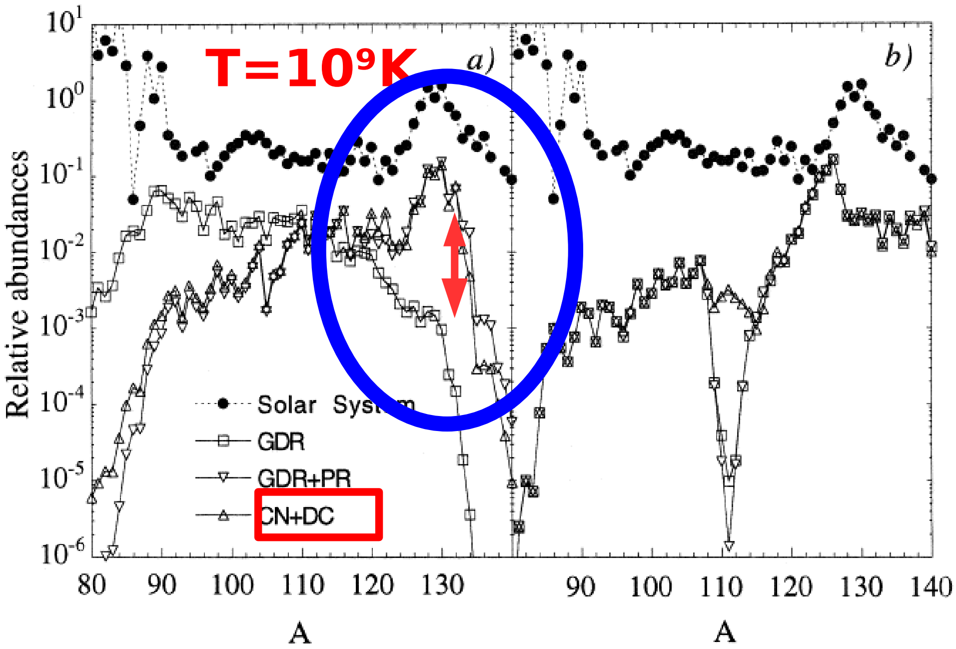
PDR= Pygmy Dipole Resonance



(nature of PDR still under debate)

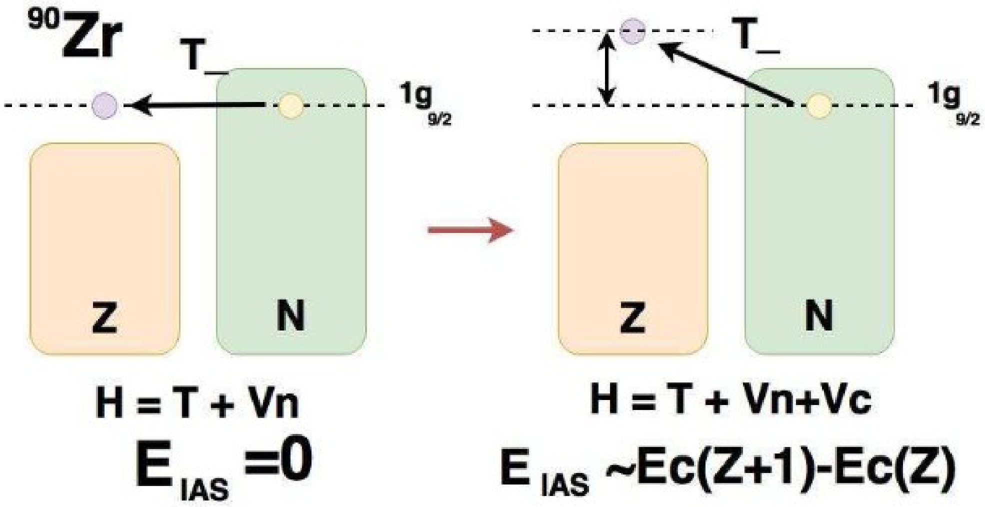
Radiative neutron captures by neutron-rich nuclei and the r-process nucleosynthesis

S. Goriely, Phys.Lett.B 436 (1998) 10-18



Low energy dipole strength in neutron-rich nuclei influences the neutron capture cross section and, thus, the r-process nucleosynthesis

From Earth: Isobaric Analog State and the breaking of isospin symmetry



- **Analog state** can be defined: $|A\rangle = \frac{T_-|0\rangle}{\langle 0|T_+T_-|0\rangle}$
- **Displacement energy or** E_{IAS}

$$E_{IAS} = E_A - E_0 = \langle A|\mathcal{H}|A\rangle - \langle 0|\mathcal{H}|0\rangle = \frac{\langle 0|T_+[\mathcal{H},T_-]|0\rangle}{\langle 0|T_+T_-|0\rangle} = \frac{m_1}{m_0}$$

$E_{IAS} \neq 0$ **only due to Isospin Symmtry Breaking terms** $\mathcal{H}$

E^{exp}_{IAS} **usually accuratelly measured !**

From Earth: Isobaric Analog State and the breaking of isospin symmetry

→ Coulomb direct contribution: a simple model

- Assuming indepentent particle model and good isospin for $|0\rangle$ $(\langle 0|T_+T_-|0\rangle = 2T_0 = N - Z)$

$$E_{IAS} \approx E_{IAS}^{C,direct} = \frac{1}{N-Z}\int [\rho_n(\vec{r}) - \rho_p(\vec{r})]\, U_C^{direct}(\vec{r})\, d\vec{r}$$

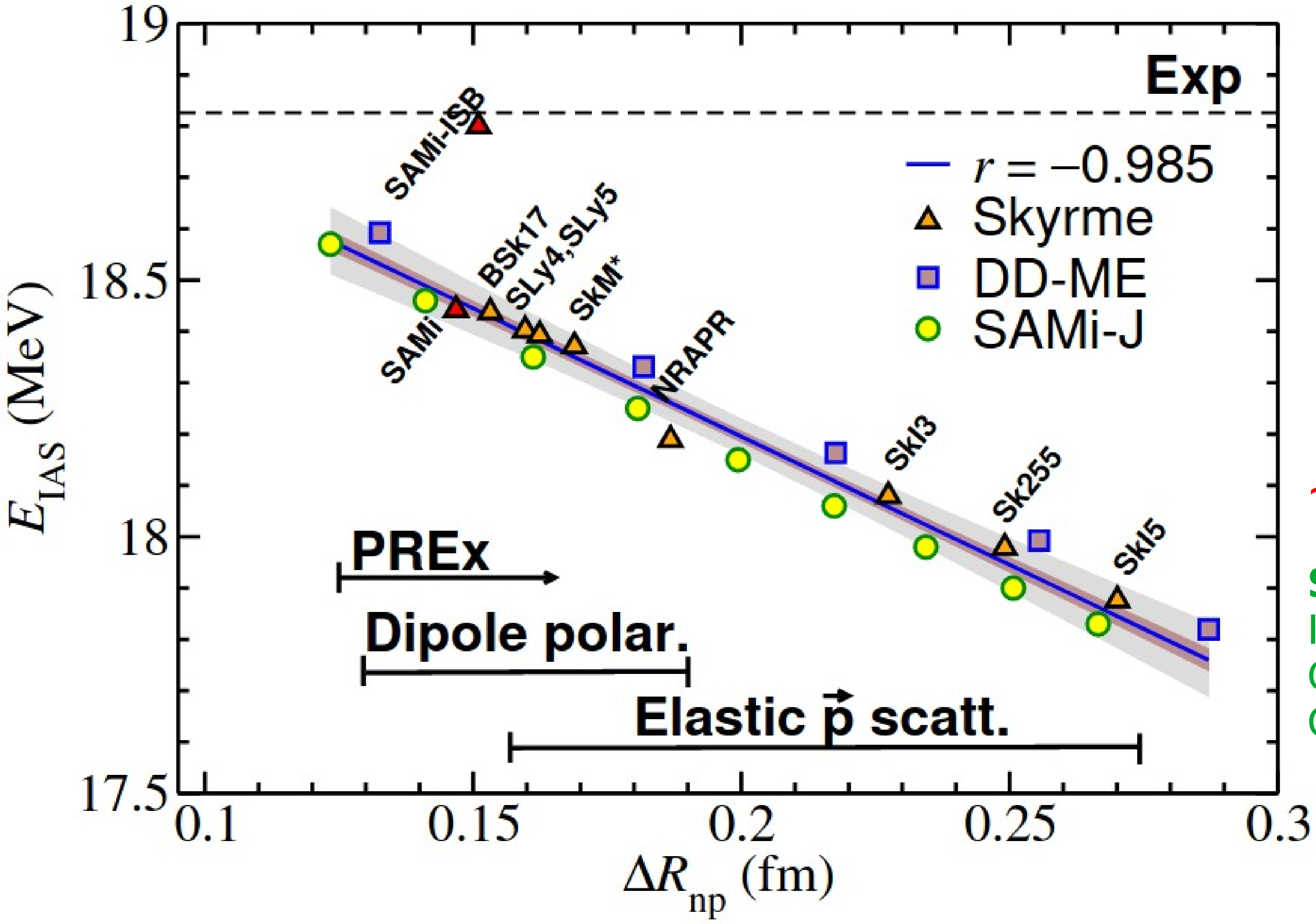
where $U_C^{direct}(\vec{r}) = \int \frac{e^2}{|\vec{r}_1 - \vec{r}|}\rho_{ch}(\vec{r}_1)\, d\vec{r}_1$

- Assuming also a uniform neutron and proton distributions of radius R_n and R_p respectively, and $\rho_{ch} \approx \rho_p$ one can find

$$E_{IAS} \approx E_{IAS}^{C,direct} \approx \frac{6}{5}\frac{Ze^2}{R_p}\left(1 - \sqrt{\frac{5}{12}}\frac{N}{N-Z}\frac{\Delta r_{np}}{R_p}\right)$$

One may expect: **the larger the Δr_{np} the smallest E_{IAS}**

From Earth: Isobaric Analog State and the breaking of isospin symmetry



Exp errors in IAS ~ tens of keV

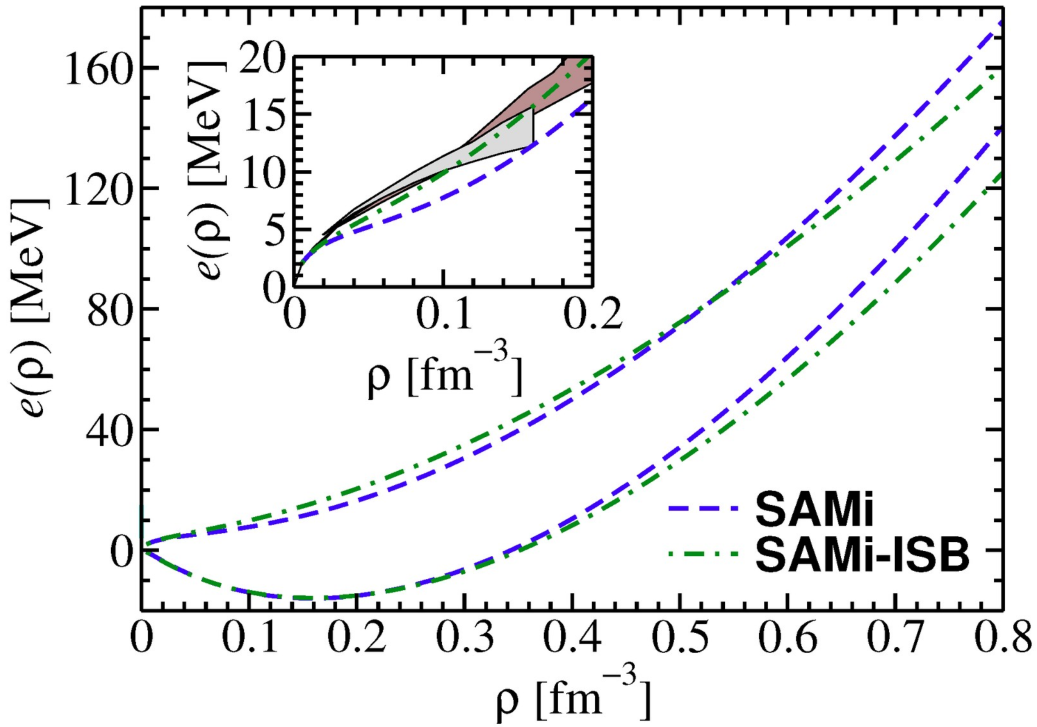
(or smaller)

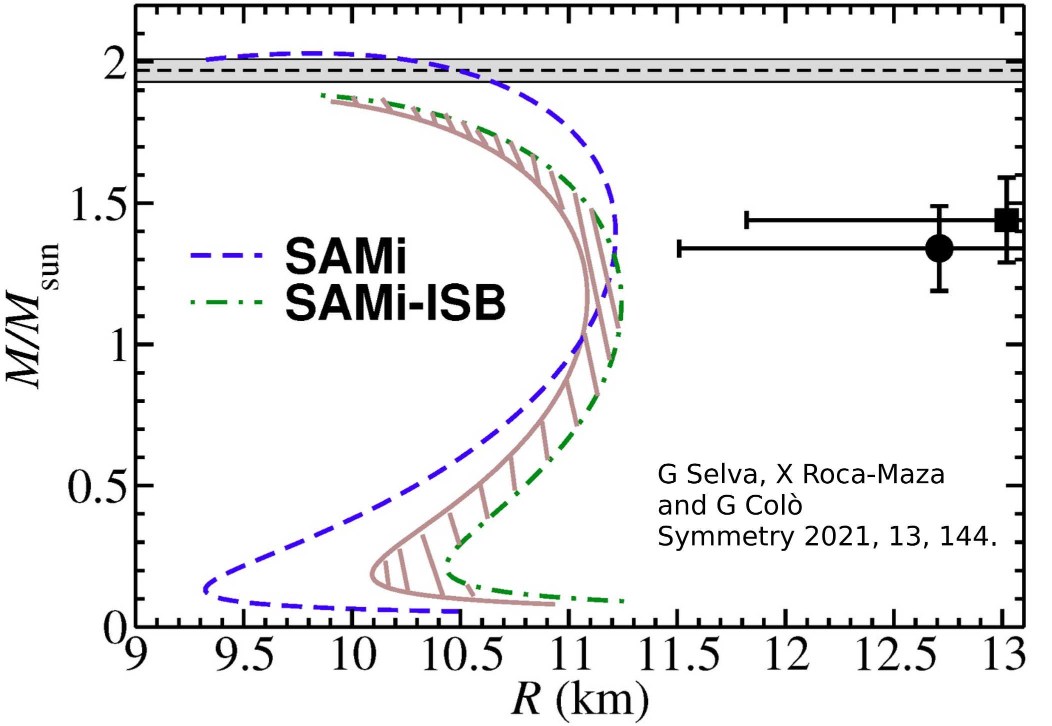
Exp width IAS

~ hundreds of keV

SAMi-ISB: includes ISB effects due to Coulomb exchange, QED corrections and nuclear ISB

From Heaven: ISB effects on NS?
(very much speculative)

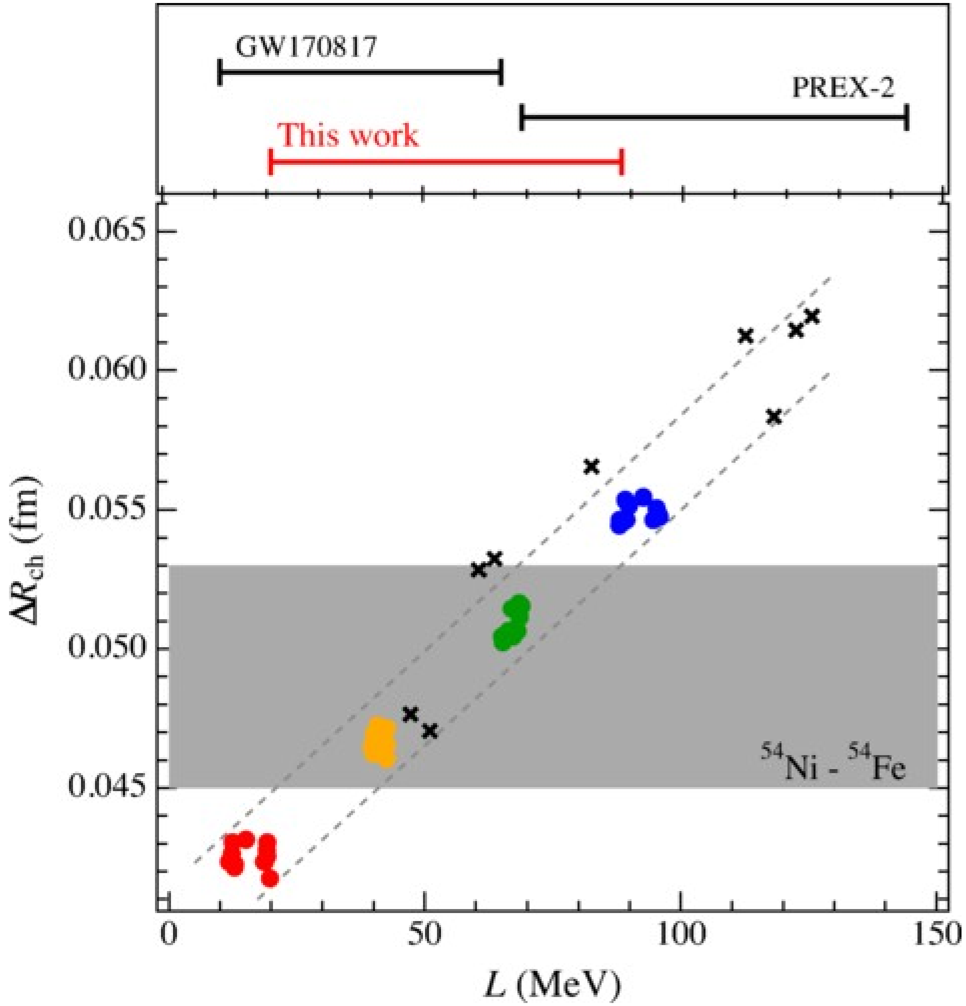




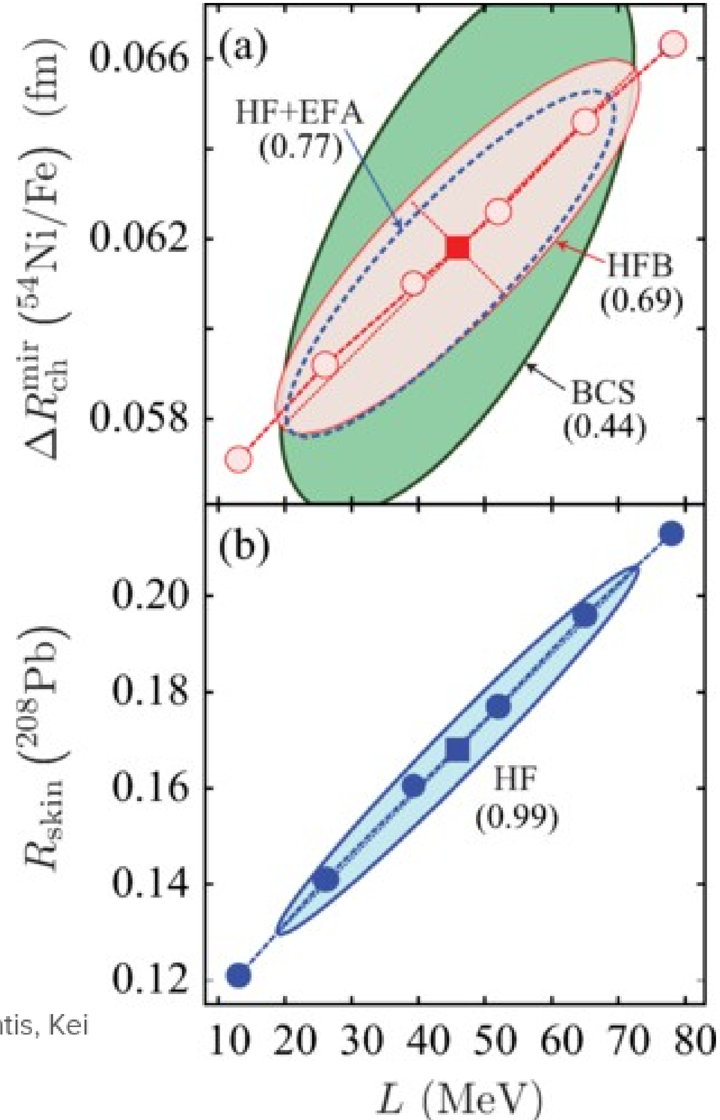
	M_{max}/M_{sun}	R_{max} [km]	$\rho^c_{1.4}$ [fm^{-3}]	$R_{1.4}$	$\Lambda_{1.4}$ [km]	$\xi_{1.4}$
SAMi	2.03	9.8	0.54	11.2	301	0.18
SAMi-ISB	1.88	9.8	0.59	11.2	261	0.19
SAMi-ISB ($u_0 = s_0 = 0$)	1.86	9.9	0.61	11.0	242	0.19

From Earth: charge radii difference in mirror mass nuclei

Isospin symmetry → $\Delta r_{ch} := r_{ch}(^{54}Ni) - r_{ch}(^{54}Fe) = \Delta r_{np}(^{54}Fe)$

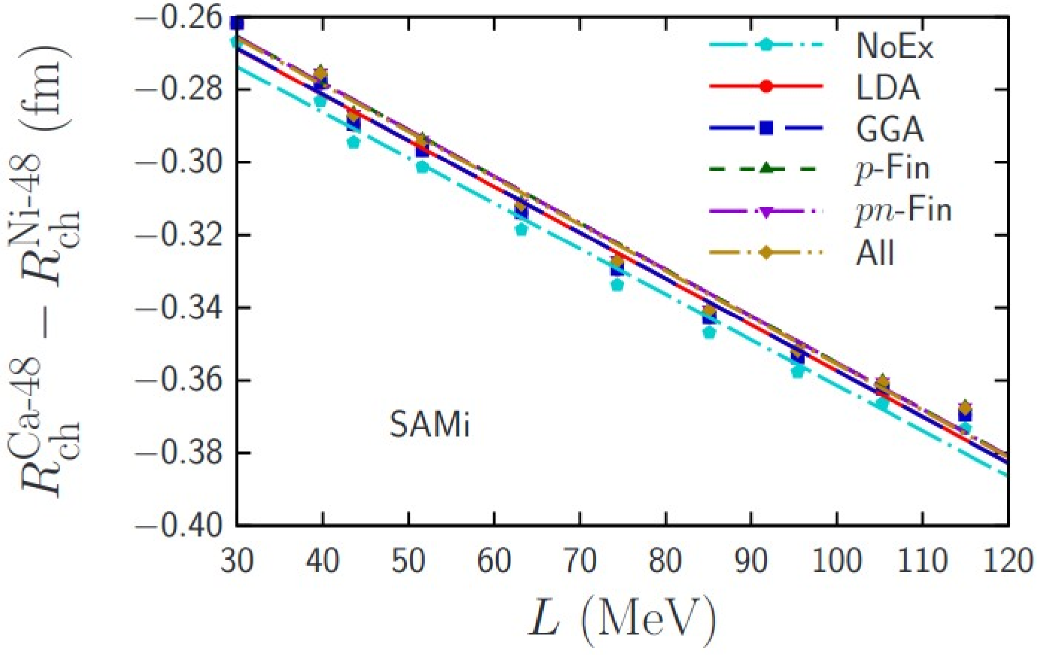


Skyy V. Pineda, Kristian König, Dominic M. Rossi, B. Alex Brown, Anthony Incorvati, Jeremy Lantis, Kei Minamisono, Wilfried Nörtershäuser, Jorge Piekarewicz, Robert Powel, and Felix Sommer
Phys. Rev. Lett. **127**, 182503 – Published 29 October 2021



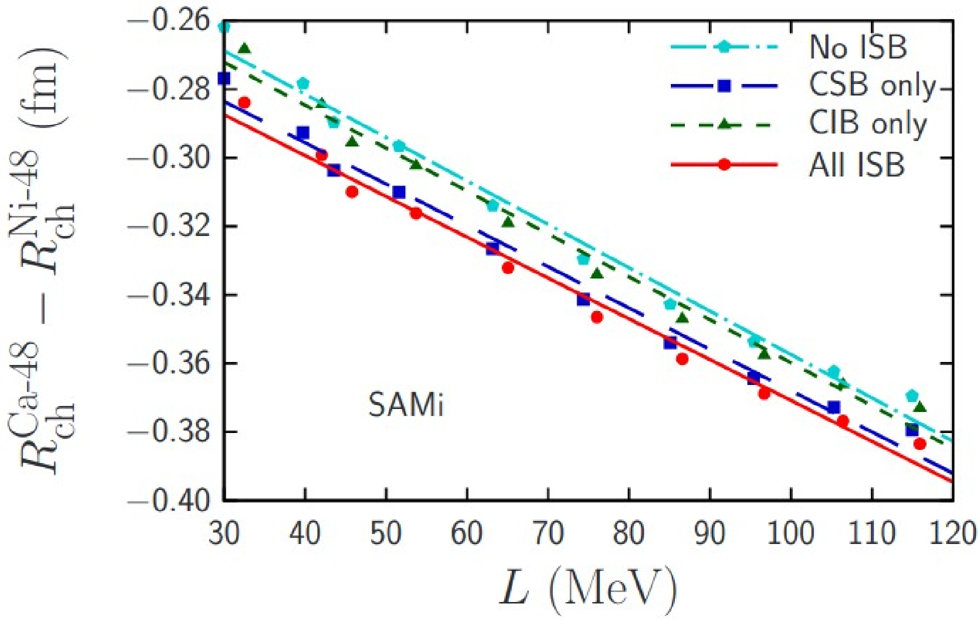
Paul-Gerhard Reinhard and Witold Nazarewicz
Phys. Rev. C **105**, L021301 – Published 3 February 2022

From Earth: charge radii difference in mirror mass nuclei – ISB



→ Accurate treatment of **Coulomb** (leading ISB in nuclei).

→ **No large effects found**.



→ **Nuclear ISB** effect may impact on ***L*** determination by about **10 MeV** (SAMi-ISB)

→ Theoretical uncertainties must be estimated. Little knowledge about ISB in the medium

Tomoya Naito, Xavier Roca-Maza, Gianluca Colò, Haozhao Liang, Hiroyuki Sagawa, arXiv:2202.05035

Summary from Progress in Particle and Nuclear Physics 101 (2018) 96–176

EoS par.	Observable	Range	Comments
ρ_0	$\langle r_{ch}^2\rangle^{1/2}$	0.154–0.159	Most accurate EDFs on $M(N,Z)$ and $\langle r_{ch}^2\rangle^{1/2}$ (see Section 5)
e_0	$M(N,Z)$	-16.2 to -15.6	Most accurate EDFs on $M(N,Z)$ and $\langle r_{ch}^2\rangle^{1/2}$ (see Section 5)
K_0	$M(N,Z)$	220–245	Most accurate EDFs on $M(N,Z)$ and $\langle r_{ch}^2\rangle^{1/2}$ (see Section 5)
	ISGMR	220–260	From EDFs in closed shell nuclei [116]
	ISGMR	250–315	Blaizot's formula [Eq. (32)] [51]
	ISGMR	$\sim$200	EDF describing also open shell nuclei [118]
J	$M(N,Z)$	29-35.6	Most accurate EDFs on $M(N,Z)$ and $\langle r_{ch}^2\rangle^{1/2}$ (see Section 5)
	IVGDR	$\sim$24.1(8) + L/8	From EDF analysis [$S(\rho = 0.1\ \text{fm}^{-3}) = 24.1(8)$ MeV] [273]
	PDS	30.2–33.8	From EDF analysis [370]
	PDS	31.0–33.6	From EDF analysis [371]
	α_D	$24.5(8) + 0.168(7)L$	From EDF analysis ^{208}Pb [96]
	α_D	30–35	From EDF analysis [179]
	IAS and Δr_{np}	30.2–33.7	From EDF analysis [325]
	AGDR	31.2–35.4	From EDF analysis [401]
	PDS, α_D, IVGQR, AGDR	32–33	From EDF analysis [508]
	compilation	29.0–32.7	[106]
	compilation	30.7–32.5	[107]
	compilation	28.5–34.9	[3]
L	$M(N,Z)$	27–113	Most accurate EDFs on $M(N,Z)$ $\langle r_{ch}^2\rangle^{1/2}$ (see Section 5)
	ρ_n	40–110	proton-^{208}Pb scattering [24]
	ρ_n	0–60	π photoproduction (^{208}Pb) [181]
	ρ_n	30–80	antiprotonic at. (EDF analysis) [102,509]
	ρ_{weak}	>20	Parity violating scattering [27]
	PDS	32–54	From EDF analysis [370]
	PDS	49.1–80.5	From EDF analysis [371]
	α_D	20–66	From EDF analysis [179]
	IVGQR and ISGQR	19–55	From EDF analysis [101]
	IAS and Δr_{np}	35–75	From EDF analysis [325]
	AGDR	75.2–122.4	From EDF analysis [401]
	PDS, α_D, IVGQR, AGDR	45.2–54.6	From EDF analysis [508]
	compilation	40.5–61.9	[106]
	compilation	42.4–75.4	[107]
	compilation	30.6–86.8	[3]

Some alternative compilations:

M. Oertel, M. Hempel, T. Klähn, S. Typel, Rev. Modern Phys. 89 (2017) 015007.

M.B. Tsang, et al., Phys. Rev. C 86 (2012) 015803.

Bao-An Li, Xiao Han, Phys. Lett. B 727 (1) (2013) 276–281.

Summary

with qualitative indication of accuracy needed to describe experiment (note that absolute values might be subject to systematics)

→ $\rho_0 \in$ [0.154,0.159] fm^{-3} → relative accuracy 2%

→ needed to describe experiment (Rch) ≤0.1%

→ $e_0 \in$ [15.6,16.2] MeV→ relative accuracy 4%

→ needed to describe experiment (B) ≤0.0001%

→ $K_0 \in$ [200,260] MeV→ relative accuracy 25%

→ needed to describe experiment (E_x^{GMR}) ≤7%

→ J ∈ [30,35] MeV → relative accuracy (α) 15%

→ needed to describe experiment ≤15%

→ L ∈ [20,120] MeV → relative accuracy (α) 150%

→ needed to describe experiment ≤50%

→ ...